Int. J. Solids Structures Vol. 29, No. 10, pp. 1221–1231, 1992
Printed in Great Britain.
0020–7683/92 $5.00+ .00
Pergamon Press plc

ELIMINATION/MINIMIZATION OF EDGE-INDUCED STRESS SINGULARITIES IN FIBER COMPOSITE LAMINATES†

RICHARD M. CHRISTENSEN
Department of Applied Science and Materials Division, University of California, Lawrence Livermore National Laboratory, Livermore, CA 94550, U.S.A.

and

STEVEN J. DETERESA
Materials Division, University of California, Lawrence Livermore National Laboratory, Livermore, CA 94550, U.S.A.

(Received 13 May 1991; in revised form 5 September 1991)

Abstract—The stress singularities along the edges of a linear elastic laminate consisting of 0° and off-axis plies in uniaxial stress are shown to either vanish or be minimized by a special fiber orientation for the off-axis plies. The particular orientation is obtained analytically using both a recently proposed specific three-dimensional constitutive equation and the more general constitutive theory, but for fiber-dominated lamina. The prescribed angle is defined by the simple relationship:

$$\theta^* = \tan^{-1}\left(\frac{1}{\sqrt{\nu_L}}\right),$$

where ν_L is the longitudinal Poisson's ratio of the lamina. Tensile test data are consistent with the theoretical result.

INTRODUCTION

Continuous fiber composites are often used as one-dimensional load-bearing members. Superficially this is an ideal application since these composites have outstanding properties only in the fiber direction. The danger with using fiber composites in a completely aligned form is that the other directions are weak and even a slight disturbance can initiate failure in a transverse mode. The traditional remedy for this problem is to laminate the composite such that most of the plies are oriented in the primary load-bearing direction with a few others oriented off this direction to prevent transverse failure. Unfortunately, this lamination procedure cures one problem at the expense of introducing another.

Restricting attention to flat laminates as opposed to structures having closed cross-sections, the new problem that arises is that of the edge effect. There are no edge stresses for an isotropic material specimen with rectangular cross-section in a state of uniaxial stress. The situation is entirely different for a laminated composite. With plies having various fiber orientations, a new stress state is created along a traction-free edge. The stresses in each ply which would exist if there were no edge must be equilibrated between adjacent layers at a traction-free edge. These stresses are fiber-dominated and consequently are large in magnitude. Relative to the applied uniaxial stress they are not negligible. The self-equilibrated state is a boundary layer effect confined to the vicinity of the edge. The large magnitude stresses which must be transmitted between adjacent plies of different fiber orientation must be sustained by the matrix interface between plies and are elastically singular at the edge. Even though the generated stress state is fiber-dominated, the possible failure mode due to the interlaminar stresses is matrix-dominated. This is the worst combination of both effects.

†Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

The purpose of the present work is to outline a specific and practical means for determining fiber orientation angles which eliminate or minimize the edge effect stress singularities in linear-elastic composite laminates loaded in uniaxial stress.

CONSTITUTIVE EQUATIONS

The lamina level constitutive equation to be used in most of the following analysis is the form given by:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij}+(E_L-E)m_im_jm_km_l\varepsilon_{kl} \tag{1}$$

where:

$$m_1 = \cos\theta$$
$$m_2 = \sin\theta$$
$$m_3 = 0,$$

with θ being the angle from the axis 1 to the fiber direction and with the plies in the 1–2 plane. This constitutive equation, developed by Christensen (1988) and Christensen and Zywicz (1990), has properties in (1) given by either

$$\begin{aligned} v &= v_L \\ E &= \frac{(1-v^2)E_T}{1-v^2E_T/E_L} \\ \mu &= \frac{E}{2(1+v)} \\ \lambda &= \left(\frac{2v}{1-2v}\right)\mu, \end{aligned} \tag{2a}$$

or

$$\begin{aligned} v &= v_L \\ \mu &= \frac{1}{6}\left[\frac{3(1-v)E_T}{2(1-v^2E_T/E_L)}+2\mu_L+\mu_T\right] \\ E &= 2(1+v)\mu \\ \lambda &= \left(\frac{2v}{1-2v}\right)\mu, \end{aligned} \tag{2b}$$

with E_L, E_T, v_L, μ_L and μ_T being the five usual transversely isotropic properties determined with respect to the longitudinal (L) and transverse (T) directions of the lamina. The form (1) was developed by separating physical effects into fiber-dominated versus matrix-dominated characteristics and the two forms in (2) will be differentiated shortly.

The corresponding laminate constitutive equation was shown (Christensen and Zywicz, 1990) to be given by:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij}+2\mu\varepsilon_{ij}+\frac{(E_L-E)}{N}\left(\sum_{n=1}^{N} m_i^{(n)}m_j^{(n)}m_k^{(n)}m_l^{(n)}\varepsilon_{kl}\right), \tag{3}$$

where

$$m_1^{(n)} = \cos\theta_n$$
$$m_2^{(n)} = \sin\theta_n$$
$$m_3^{(n)} = 0,$$

and where N is the total number of plies.

The tensor form (3) for the laminate can be converted to an explicit engineering property form. The form (3) applies to any lay-up; however, the application immediately below will be restricted to orthotropic lay-ups, implying that $+\theta$ direction plies are always accompanied by corresponding $-\theta$ plies. Let

$$\Sigma_c = \frac{1}{N}\left(\sum_{n=1}^{N} \cos^4\theta_n\right)$$
$$\Sigma_s = \frac{1}{N}\left(\sum_{n=1}^{N} \sin^4\theta_n\right)$$
$$\Sigma_{cs} = \frac{1}{N}\left(\sum_{n=1}^{N} \cos^2\theta_n \sin^2\theta_n\right). \qquad (4)$$

The orthotropic laminate engineering constants are then derived from (3) and are found to be given by:

$$\nu_{12} = \frac{\nu + (1-\nu^2)\left(\frac{E_L}{E} - 1\right)\Sigma_{cs}}{1 + (1-\nu^2)\left(\frac{E_L}{E} - 1\right)\Sigma_s}$$

$$\nu_{21} = \frac{\nu + (1-\nu^2)\left(\frac{E_L}{E} - 1\right)\Sigma_{cs}}{1 + (1-\nu^2)\left(\frac{E_L}{E} - 1\right)\Sigma_c}$$

$$\nu_{13} = \nu\left(\frac{1-\nu_{12}}{1-\nu}\right)$$

$$\nu_{23} = \nu\left(\frac{1-\nu_{21}}{1-\nu}\right)$$

$$\frac{E_{11}}{E} = \frac{(1-\nu\nu_{12})}{(1-\nu^2)} + \left(\frac{E_L}{E} - 1\right)(\Sigma_c - \nu_{12}\Sigma_{cs})$$

$$\frac{E_{22}}{E} = \frac{(1-\nu\nu_{21})}{(1-\nu^2)} + \left(\frac{E_L}{E} - 1\right)(\Sigma_s - \nu_{21}\Sigma_{cs})$$

$$\nu_{31} = \frac{\nu}{\frac{\nu^2}{(1-\nu)}\left[1 + \left(\frac{\nu_{12}}{\nu_{21}}\right)\left(\frac{\nu_{23}}{\nu_{13}}\right)\right] + \frac{(1+\nu)(1-2\nu)}{(1-\nu_{12})}\left(\frac{E_{11}}{E}\right)}$$

$$\nu_{32} = \frac{\nu}{\frac{\nu^2}{(1-\nu)}\left[1 + \left(\frac{\nu_{21}}{\nu_{12}}\right)\left(\frac{\nu_{13}}{\nu_{23}}\right)\right] + \frac{(1+\nu)(1-2\nu)}{(1-\nu_{21})}\left(\frac{E_{22}}{E}\right)}$$

$$\frac{E_{33}}{E} = \frac{(1-\nu)-\nu(\nu_{31}+\nu_{32})}{(1+\nu)(1-2\nu)}$$

$$\frac{\mu_{12}}{\mu} = 1+2(1+\nu)\left(\frac{E_L}{E}-1\right)\Sigma_{cs}$$

$$\mu_{23} = \mu_{13} = \mu. \tag{5}$$

The lamina constitutive relation (1) with either of (2), the laminate form (3) and the orthotropic engineering properties, eqns (5) are exact when the following two relations are satisfied by the five transversely isotropic lamina properties:

$$\mu_L = \mu_T$$

and

$$\mu_T = \frac{(1-\nu_L)E_T}{2(1-\nu_L^2 E_T/E_L)}. \tag{6}$$

When eqns (6) are satisfied, the laminate properties (5) are identical to the properties predicted by the exact lamination procedure given by Pagano (1974). When relations (6) are not satisfied, the constitutive equations and engineering properties are based on approximations of only the matrix-dominated properties and the two different forms in (2) are two slightly different approximations. The fiber-dominated properties E_L and ν_L are not approximated in any manner.

ANGLE-PLY LAMINATE BEHAVIOR

Consider an orthotropic laminate composed exclusively of equal numbers of plies oriented at $\pm\theta$ angles, the so-called "angle-ply" laminate. The transverse Poisson's ratio is given by the engineering constants (5). The relation for ν_{12} is then:

$$\nu_{12} = \frac{\nu+(1-\nu^2)\left(\frac{E_L}{E}-1\right)\cos^2\theta\sin^2\theta}{1+(1-\nu^2)\left(\frac{E_L}{E}-1\right)\sin^4\theta}. \tag{7}$$

This result, and the corresponding through-thickness Poisson's ratio ν_{13} from (5), shows the same dependence on θ as that discussed by Herakovich (1984, 1989). The result (7) exhibits an interesting behavior. If the form

$$\cos^2\theta\sin^2\theta = \nu\sin^4\theta, \tag{8}$$

is substituted into the numerator of (7) then the whole expression reduces to:

$$\nu_{12} = \nu. \tag{9}$$

The interpretation of (9) is that the angle-ply laminate has a transverse Poisson's ratio ν_{12} equal to the longitudinal Poisson's ratio of the lamina $\nu = \nu_L$ if (8) is satisfied. The appropriate angle from (8) is given by:

$$\tan^2\theta = \frac{1}{\nu}. \tag{10}$$

Additionally, at this same angle the through-thickness Poisson's ratio ν_{13} is also equal to

ν_L. When (10) is satisfied, the angle-ply lay-up has apparently compatible behavior with a 0° lamina under a state of uniaxial stress. This possibility for compatible deformation between plies will be examined in detail in the following derivation where the implied restriction to orthotropy will be relaxed, and shear coupling effects will be explicitly considered.

EDGE SINGULARITY PROBLEM

It is common knowledge that a general laminate consisting of any proportion of plies oriented at angles θ from -90° to 90° will produce edge singularities in the linear elastic stresses when the laminate is in a state of uniaxial stress (tension or compression). These free edge-induced stresses have been extensively studied over the past 25 years beginning with the pioneering works of Hayashi (1967) and Pipes and Pagano (1970). It is now well known that edge singularities arise from two property mismatches between plies: differences in Poisson's ratios leading to in-plane transverse stresses and differences in shear coupling leading to in-plane shear stresses. Indeed, it has been stated that minimization of these engineering property differences between plies attenuates the edge stresses (Herakovich, 1981). However, other than calculating the dependence of both in-plane transverse and shear stresses on the ply angle for every material system and lay-up of interest, there is currently no analytical approach for choosing ply orientations to minimize both of these effects simultaneously. In fact, the primary focus of previous work has been on optimizing the stacking sequence of a fixed lay-up configuration, to minimize interlaminar shear stresses and to generate compressive through-thickness stresses at the edge (Herakovich, 1981). This solution to the edge problem suffers when loads are reversed and through-thickness stresses become tensile. Other methods suggested by Kim (1989) to reduce the effects of edge stress singularities include reinforcement of the free edges and hybridization of materials, but apparently no approach based solely on optimizing fiber orientation to reduce all the edge effects has been proposed.

The goal in the present work is to find ply orientations for which the edge singularities can be forced to vanish, thereby yielding an expected improvement in mechanical performance. The approach will first utilize an analysis based on the constitutive relation (1), and then the more general theory will be examined.

Consider the 0° plies which are part of the laminate. Subject to later verification, it is assumed these plies have the same stress state as would exist if the laminate were composed entirely of 0° plies, i.e.

$$\sigma_{11} = \hat{\sigma}$$

$$\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0, \tag{11}$$

and

$$\varepsilon_{11} = 1$$

$$\varepsilon_{22} = \varepsilon_{33} = -\nu_L$$

$$\varepsilon_{12} = \varepsilon_{23} = \varepsilon_{13} = 0, \tag{12}$$

where axis 1 is the direction of the uniaxial stress. The strain in direction 1 is normalized to unity for convenience; thus from (1) the stress in the 1-direction becomes:

$$\hat{\sigma} = E_L. \tag{13}$$

Now consider one off-axis ply oriented at an angle θ. For this ply to exist in geometric compatibility with the 0° ply in the laminate it must have the same strain state for ε_{11}, ε_{22} and ε_{12} (12), rewritten with $\nu = \nu_L$ for use with the constitutive relation (1). Then for the angled ply take

$$\varepsilon_{11} = 1$$
$$\varepsilon_{22} = \varepsilon_{33} = -\nu$$
$$\varepsilon_{12} = \varepsilon_{23} = \varepsilon_{13} = 0. \tag{14}$$

With the strains (14), the dilatation is given by:

$$\varepsilon_{kk} = 1-2\nu, \tag{15}$$

and substituting (14) and (15) into the constitutive relation (1) gives the two components of in-plane stress which cause edge effects:

$$\sigma_{22} = (1-2\nu)\lambda-2\nu\mu+(E_L-E)[\sin^2\theta\cos^2\theta-\nu\sin^4\theta]$$
$$\sigma_{12} = (E_L-E)[\sin\theta\cos^3\theta-\nu\sin^3\theta\cos\theta], \tag{16}$$

along with a similar expression for the stress in the load direction σ_{11}. Using λ from either of (2) in (16), these two stresses become:

$$\sigma_{22} = (E_L-E)\sin^2\theta[\cos^2\theta-\nu\sin^2\theta]$$
$$\sigma_{12} = (E_L-E)\sin\theta\cos\theta[\cos^2\theta-\nu\sin^2\theta]. \tag{17}$$

The two stress components in (17) vanish if $\theta = \theta^*$ where:

$$\tan^2\theta^* = \frac{1}{\nu} = \frac{1}{\nu_L}. \tag{18}$$

With (18) the complete state of stress from (1) in the θ lamina is:

$$\sigma_{11} \neq 0$$
$$\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0. \tag{19}$$

The same state of stress exists for a $-\theta$ lamina if (18) is satisfied. Thus the three-dimensional states of strain (14) and stress (19) in the θ (+ or −) plies are completely compatible with the three-dimensional states of strain (12) and stress (11) in the 0° ply. The absence of stress components σ_{22} and σ_{12} in the θ plies precludes the existence of edge stress singularities in laminates composed solely of 0° and θ plies. Any lay-up angle other than that defined by (18) will require the presence of stress singularities along the edges of the laminate.

To summarize, for a laminate governed by the linear elastic constitutive equation (1) and loaded in uniaxial stress in the 1-direction, there will be no edge stress singularities along the lateral edges of any of the laminas if the 0° plies are combined with off-axis plies having the prescribed orientation:

$$\theta^* = \tan^{-1}\left(\frac{1}{\sqrt{\nu_L}}\right), \tag{20}$$

where ν_L is the longitudinal Poisson's ratio of the basic lamina. This condition is true for any proportion and any stacking sequence (orthotropy is not required) of the 0° and $\pm\theta^*$ plies. The absence of edge stresses in these laminates is due to the fact that the off-axis ply under uniaxial stress deforms exactly as the 0° lamina, exhibiting the same Poisson's ratio and no shear coupling. Consequently, there is no warpage or twisting in unbalanced and unsymmetrical versions of these laminates.

For relevant values of ν_L the angle θ^* is very close to 60°, e.g.

$$\nu_L = \tfrac{1}{3}, \quad \theta^* = 60^\circ$$
$$\nu_L = 0.3, \quad \theta^* = 61.3^\circ. \tag{21}$$

The result (20) is exact from the point of view of three-dimensional linear elasticity theory down to and including the lamina level when the two matrix-dominated property relations (6) are satisfied. However, even when they are not satisfied, it will be shown next that the entire procedure is still applicable as a reasonable approximation.

For composite systems not modeled by the constitutive equation (1), the constitutive form of classical lamination theory is used as the starting point. These plane stress forms can be taken as trial forms for the three-dimensional solution of interest here. Proceeding with the same uniaxial stress formulation, now restricted to an orthotropic laminate, the transverse stress σ_{22} and shear stress σ_{12} for the $\pm\theta$ plies and the transverse stress σ_{22} for the 0° ply are all set equal to zero. The system of resulting equations is overdetermined, comprising three equations in two unknowns ε_{22} and $\tan^2\theta$. However, important information can be extracted from the system. The equations contain terms of order one, $O(1)$, and of order, $O(E_T/E_L)$. For a fiber-dominated system the latter terms are small compared with the former ones. Accordingly, the system of equations can be set up as a standard perturbation expansion by taking

$$\tan^2\theta = \alpha_0 + \alpha_1(E_T/E_L) + \alpha_2(E_T/E_L)^2 + \cdots$$
$$\varepsilon_{22} = \beta_0 + \beta_1(E_T/E_L) + \beta_2(E_T/E_L)^2 + \cdots, \tag{22}$$

where the α_is and β_is are to be determined. Substituting these expansions into the system of three equations allows a reduction to two equations which differ in the $O(E_T/E_L)$ terms. Neglecting the terms $O(E_T/E_L)$, then to a first-order approximation the system reduces to:

$$1 - \nu_L\alpha_0 = 0, \tag{23}$$

which gives the same result for θ^* found previously in (20). Therefore, when a composite material is fiber-dominated, the orientation angle (20) still provides the solution that satisfies the edge condition. In this more general case the stresses, σ_{22} and σ_{12}, which yield edge effects do not vanish, rather they have magnitudes that are characteristic of matrix-dominated properties. In this sense, the in-plane stresses that cause the edge singularities are minimized with respect to fiber-dominated effects (e.g. the applied uniaxial stress). The difference between the fiber-dominated and matrix-dominated stresses can be as much as an order of magnitude.

Calculations of transverse normal and in-plane shear stresses in $[0_n/\pm\theta]_s$ laminates under uniaxial stress were made using classical lamination theory. The magnitude of the lamina edge traction in the off-axis plies is given by:

$$\sigma_{edge} = \sqrt{\sigma_{22}^2 + \sigma_{12}^2}. \tag{24}$$

This stress and the 0° ply traction must be equilibrated to zero through a superposed edge stress giving rise to the edge singularities. For common material systems based on carbon, glass, boron and aramid fibers, these tractions reached minimum or vanishing values near the respective θ^* values. An example of how the edge tractions vary with angle in a $[0_6/\pm\theta]_s$ laminate of AS4/3501-6 (Hercules Inc.) carbon/epoxy is shown in Fig. 1. These calculations were based on lamina properties reported by Kim *et al.* (1988).

Of the two derivatives leading to the prescribed angle (20), the one based on the constitutive equation (1) is the stronger. It does not require $(E_T/E_L) \ll 1$ or orthotropy, and it leads to an identical satisfaction of the traction-free edge condition without the presence of edge singularities. The latter derivation is less general in these particulars and does not admit the limiting case of an angle-ply laminate.

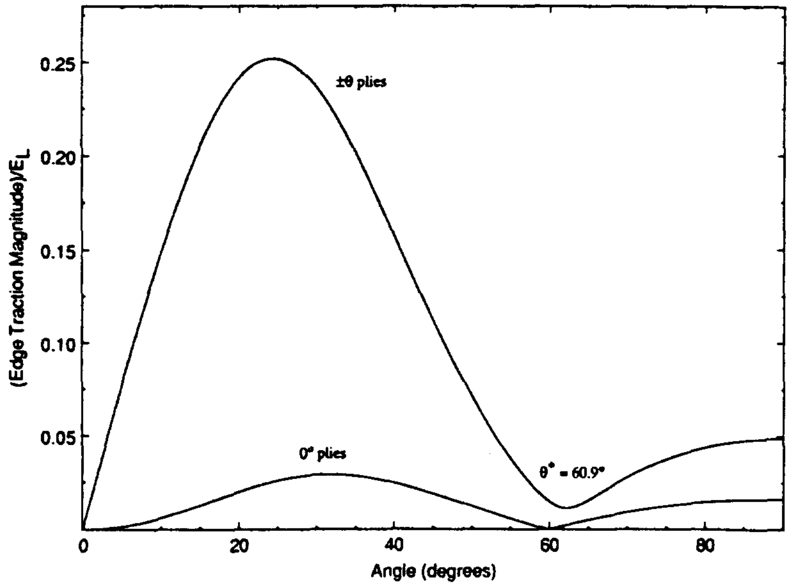

Fig. 1. Normalized edge tractions in $[0_N/\pm\theta]_s$ laminates of AS4/3501-6 under uniaxial stress σ_{11} with $\varepsilon_{11} = 1$.

EXPERIMENTAL RESULTS

In a study of the contribution of free-edge interlaminar stresses to laminate uniaxial tensile failure, Sun and Zhou (1988) determined the dependence of failure loads on load direction for $[0/60/\text{-}60]_s$ quasi-isotropic laminates. A single panel of AS4/3501-6 was used to machine specimens for testing along 0°, 10°, 20° and 30° directions measured with respect to the 0° plies. Following the analysis presented here and using a reported value of 0.3 (Kim *et al.*, 1988; Sun and Zhou, 1988) for the lamina longitudinal Poisson's ratio of this composite material, the critical angle for minimization of interlaminar stresses is calculated from (20) to be 60.9°. Consequently, the particular quasi-isotropic lay-up selected for these tests should be free of significant interlaminar stresses only when tested along the 0° direction.

The highest strengths measured by Sun and Zhou (1988) were in the 0° tests. It was ascertained that failure along this direction was unaffected by interlaminar stresses and was adequately predicted using classical lamination theory to determine ply failure. Strengths of specimens tested off-axis at 10°, 20° and 30° were found to be approximately half those measured along the 0° direction. This degradation in strength was attributed to failure due to the interlaminar stresses generated in off-axis loading. The lowest tensile strengths were obtained with the 30° test, the case in which the calculated interlaminar stresses were most severe. This test corresponds to the uniaxial loading of a $[30/90/\text{-}30]_s$ laminate. Normal through-thickness interlaminar stresses are predicted to be tensile for this stacking sequence; hence they can contribute to edge delamination. A change in the stacking sequence of the quasi-isotropic laminate comprised of 90° and ±30° plies can cause the normal interlaminar stresses to become compressive and therefore less detrimental. However, the interlaminar shear stresses can still contribute to edge failure. To determine the tensile strength of a related laminate designed to generate compressive through-thickness normal edge stresses, a small test program was conducted.

A quasi-isotropic $[0/\pm 60]_{2s}$ lay-up was used for static tensile tests in two directions: parallel and perpendicular to the 0° plies. A single panel of AS4/3501-6 was fabricated according to manufacturer's specifications and tested according to ASTM-D3039 procedures by Integrated Technologies, Inc. (Bothell, WA). Testing along the 0° direction corresponds to tests performed by Sun and Zhou (1988) along the same direction. Loads along a 90° direction are applied to a $[90/\pm 30]_{2s}$ lay-up which, in contrast to the test of a $[30/90/\text{-}30]_s$ laminate by Sun and Zhou, is expected to experience compressive through-thickness interlaminar normal stresses.

The particular lay-up $[0/\pm 60]_{2s}$ was chosen for several reasons. First, the in-plane elastic properties are independent of direction. Second, loading along directions parallel and perpendicular to the 0° plies nearly minimizes in the former case and maximizes in the latter case the interlaminar stress effects. Therefore, a single panel can be used to test the effects of ply interaction without complications due to variable composite quality as defined by fiber content, waviness and distribution, void content and degree of matrix cure. Third, because the lay-up is isotropic in thermal expansion behavior, the magnitudes of the in-plane residual stresses are equivalent in the two perpendicular test directions. However, the sign of these stresses varies between plies. For the chosen lay-up stacking sequence the 0° test exhibits transverse tension in the outer (0°) plies and the 90° test exhibits transverse compression in the outer (90°) plies. These transverse stresses promote through-thickness edge stresses which are tensile for the 0° test and compressive for the 90° test. Fourth, as previously mentioned, the stacking sequence was selected to promote through-thickness compressive edge stresses in the 90° tensile test. (The corresponding stresses in the 0° test should be negligible.) Thus it is seen that in terms of the through-thickness edge stress, the stacking sequence of the lay-up $[0/\pm 60]_{2s}$ is the worst case in the 0° test due to the residual stresses and is the best case in the 90° test due to both the thermal stresses and those produced in uniaxial tension.

The results of tensile testing five coupons each in the two perpendicular directions are summarized in Table 1. Average values of tensile moduli were identical to within 5%. This agreement is comparable to that found by Swanson and Trask (1989) for off-axis tensile loading of $[0/\pm 45/90]_s$ quasi-isotropic cylinders of AS4/3501-6 and is confirmation of the in-plane isotropy of the elastic properties of the lay-up. In contrast, tensile strength was not isotropic. The average ultimate tensile strength of the laminate along the 0° direction was 32% greater than that measured along the 90° direction. Typical stress–strain curves for each test direction are shown in Fig. 2. Both tests exhibited audible cracking in the range of 0.6–0.8% strain, which was presumably due to the transverse tensile failure of the ±60° and 90° plies in the parallel and perpendicular tests, respectively. The inability of the predominant load-bearing ±30° plies in the 90° tests to carry as much load as the 0° plies in the 0° tests is most likely due to the large in-plane shear stresses and resultant interlaminar stresses experienced by the former. Thus it is seen that despite efforts to alleviate the detrimental effects of through-thickness tensile interlaminar stresses in a 90°, ±30° laminate by a judicious choice of stacking sequence, performance in uniaxial tension is still inferior to that of a 0°, ±60° laminate.

Absolute values of in-plane tensile modulus and strength given in Table 1 are lower than values reported by Sun and Zhou (1988) and by Swanson and Trask (1989) for quasi-isotropic lay-ups of AS4/3501-6 coupons and tubes, respectively. This discrepancy is due

Table 1. Tensile test results for AS4/3501-6 quasi-isotropic lay-up $[0/\pm 60]_{2s}$

Specimen ID number	Test direction (°)	Tensile modulus (GPa)	Tensile strength (MPa)	Tensile strain at failure (%)
LL1-W-1-1	0	45.2	562	1.24
LL1-W-1-2	0	42.9	571	1.33
LL1-W-1-3	0	43.5	586	1.35
LL1-W-1-4	0	44.0	566	1.29
LL1-W-1-5	0	44.6	599	1.34
Mean	—	**44.0**	**577**	**1.31**
Coefficient of variation (%)	—	2.02	2.68	3.41
LL1-F-1-1	90	42.0	437	1.04
LL1-F-1-2	90	41.7	435	1.04
LL1-F-1-3	90	42.9	446	1.04
LL1-F-1-4	90	41.5	430	1.03
LL1-F-1-5	90	41.2	441	1.07
Mean	—	**41.9**	**438**	**1.04**
Coefficient of variation (%)	—	1.53	1.57	1.45

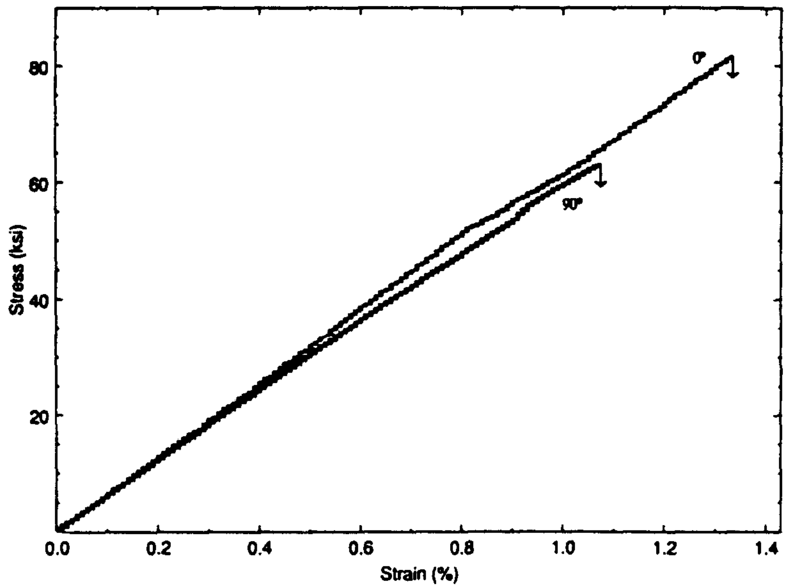

Fig. 2. Typical tensile behavior of $[0/\pm 60]_s$ laminates of AS4/3501-6 tested along 0° and 90° directions.

to a lower fiber volume fraction in the specimens tested in the present study. Cured ply thicknesses averaged 0.17 mm instead of the 0.13 mm commonly reported for this material system. Strength values can be compared by normalizing to an equivalent ply thickness. However, comparison of failure strains is a more direct approach. The average 1.31% strain at failure measured along the 0° direction is virtually identical to the fiber strains at failure measured by Swanson and Trask (1989) using tubular specimens. Tubular specimens do not suffer from the edge stress singularities since there is no free edge in the gage section. Thus, the agreement in failure strains between tubular specimens and the 0° tests in this study suggests there is little effect, if any, of interlaminar stresses on the tensile performance of the latter. This conclusion is in complete accord with that reached by Sun and Zhou (1988) for failure of $[0/60\text{-}60]_s$ laminates loaded along the 0° direction.

CONCLUSIONS

An analytical approach to eliminate or minimize edge stress singularities in fiber composite laminates via ply orientation has been proposed. Use of the prescribed lay-up in one-dimensional load-bearing composite structures should improve performance over that obtained with any other off-axis ply orientation. Although improvement in static tensile strength due to the judicious choice of fiber orientation has been demonstrated, the more important use for the analytical results of this study is in designing composite structures for fatigue loads. Elimination or minimization of the edge stress singularities should result in a structure which is more resistant to delamination and damage accumulation under cyclic loading.

Acknowledgements—R. M. Christensen is appreciative of support from the Office of Naval Research, Dr Y. Rajapakse, and S. J. DeTeresa acknowledges the generous support of the U.S. Army Ballistic Research Laboratory, Dr Bruce Burns. The authors wish to express sincere thanks to Mr Peter Nackowski of the Composites Products Group of Hercules Aerospace Co. for supplying the AS4/3501-6 prepreg tape and to Mr Greg Strand and Ms Farz Narcisso of Integrated Technologies, Inc. for their assistance in specimen fabrication and testing.

REFERENCES

Christensen, R. M. (1988). Tensor transformations and failure criteria for the analysis of fiber composite materials. *J. Comp. Mater.* **22**, 874–897.

Christensen, R. M. and Zywicz, E. (1990). A three-dimensional constitutive theory for fiber composite laminated media. *J. Appl. Mech.* **57**, 948–955.

Hayashi, T. (1967). Analytical study of interlaminar shear stresses in a laminate composite plate. *Trans. Japan Soc. Aeron. & Space Sci.* **10**(7), 43–48.

Herakovich, C. T. (1981). On the relationship between engineering properties and delamination of composite materials. *J. Comp. Mater.* **15**, 336–348.

Herakovich, C. T. (1984). Composite laminates with negative through thickness Poisson's ratios. *J. Comp. Mater.* **16**, 447–455.

Herakovich, C. T. (1989). Effects of morphology on properties of graphite composites. *Carbon* **27**(5), 663–678.

Kim, R. Y. (1989). Experimental observations of free-edge delamination. In *Interlaminar Response of Composite Materials* (Edited by N. J. Pagano). Elsevier, Amsterdam.

Kim, R. Y., Abrams, F. and Knight, M. (1988). Mechanical characterization of a thick composite laminate. *Proceedings of the American Society for Composites, 3rd Technical Conference*. Technomic, Lancaster, PA.

Pagano, N. J. (1974). Exact moduli of anisotropic laminates. In *Mechanics of Composites Materials* (Edited by G. P. Sendeckyj), Vol. 2. Academic Press, New York.

Pipes, R. B. and Pagano, N. J. (1970). Interlaminar stresses in composite laminates under uniform axial extension. *J. Comp. Mater.* **4**, 538–548.

Swanson, S. R. and Trask, B. C. (1989). Strength of quasi-isotropic laminates under off-axis loading. *Comp. Sci. Tech.* **34**, 19–34.

Sun, C. T. and Zhou, S. G. (1988). Failure of quasi-isotropic composites laminates with free edges. *J. Reinforced Plast. Comp.* **7**, 515–557.